ON THE AUTONOMOUS DETERMINATION OF POSITION OF A MOVING OBJECT BY MEANS OF A SPACE GYROCOMPASS, A DIRECTIONAL GYROSCOPE AND AN INTEGRATING DEVICE

(OB AVTONOMOM OPREDELENII MESTOPOLOZHENIIA DVIZHUSHCHEGOSIA OB'EKTA POSREDSTVOM POSTRANSTVENNOGO GIDROSKOPICHESKOGO KOMPASA, GIROSKOPA NAPRAVLENIIA I INTEGRIRUIUSHCHEGO USTROISTVA)

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A theoretical solution of the problem of the determination of position of a moving object by means of gyroscopes, accelerometers, and integrating devices was presented in [1]. The scheme of combining the computed elements as shown in [1] could be modified in many different ways. This work presents one of the variants of a somewhat different solution of the above problem with the use of the properties of Geckeler's gyrocompass. The exact theory of Geckeler's gyrocompass is given in [2]*. This variant of a solution of the problem of an autonomous determination of position of an object moving on a spherical earth requires the use of a directional gyroscope and of a device integrating a system of three nonlinear differential equations of the first order. As far as these equations are concerned, the mathematics of the problem remains in principle the same as in [1].

1. A discussion of the basic properties of the sensing element** will introduce the main subject (Fig. 1). We shall assume a spherical earth,

* In this connection the publication of Fox [3] should be mentioned, where, by means of a complicated gyroscopic system, also containing a gyrocompass, essentially the same problem is solved from the measurements of gyroscopic reactions.

** And also the sensing element of a twin vertical gyroscope described in [4].

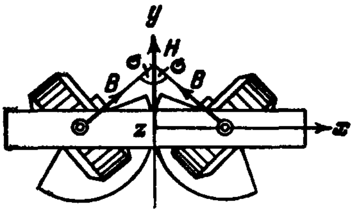

Fig. 1.

a central field of gravitation, and some initial conditions orienting the sensing element and furnishing the angle 2σ between the axes of spin of the two gyroscopes in the gyrocompass. If the suspension point of the sensing element remains on the earth's surface, then the line joining the suspension point with the center of gravity will always pass through the center of the earth. Moreover, the vector sum H of the angular momenta of the two gyroscopes of the sensing element will always remain perpendicular to the velocity vector of the suspension point moving with respect to a stationary sphere S, whose center coincides with that of the earth and which has the same radius as the earth.* Finally, the magnitude of the velocity of the suspension point v is related to the angle between the axes of spin of the sensing element in a space gyrocompass by the formula

$$2B \cos \sigma = mav \tag{1}$$

where B is the angular momentum of each gyroscope, m is the total mass of the sensing element, and a is the distance between its center of gravity and its suspension point.**

2. From the above properties of a space gyrocompass it follows that if the suspension point of the sensing element is fixed with respect to

* The introduction of such a sphere, a kind of "nonrotating earth", is very convenient and helpful in a great number of problems on the theory of gyroscopes (see [1], [2], [4], [6]).

** It can be shown that any twin gyroscopic device with its center of gravity elsewhere than in [2] or [4] possesses identical properties if: (1) the center of gravity of its sensing element is in the plane containing the suspension point which is perpendicular to the vector sum of angular momenta of the gyroscopes; (2) the distance a from the suspension point to the center of gravity is the same as in the space gyrocompass of Geckeler [2] or in the twin vertical gyroscope [4].

the earth's surface, then the vector sum of the angular momenta of the gyroscopes, H, is directed exactly North.

In this case we have

$$v = RU\cos\varphi \tag{2}$$

where ϕ is the geocentric latitude, R is the earth's radius, and U is the earth's angular velocity. From (1) and from (2) we can determine ϕ when the angle between the axes of spin of the gyroscopes is known.

When the suspension point is not fixed with respect to the earth's surface, and moves, we cannot determine the direction of a meridian and the latitude from the readings of a space gyrocompass only; we must in addition know the velocity vector of the suspension point with respect to the earth's surface.

3. We shall now consider a new variant of the theoretical solution of the problem of determining the position of a moving object. Let the origin of an orthogonal coordinate system xyz coincide with the suspension point of the sensing element, the y axis be along the vector sum H of the angular momenta of the gyroscopes, and the z-axis be along the line joining the center of gravity of the sensing element with its suspension point (Fig. 2).

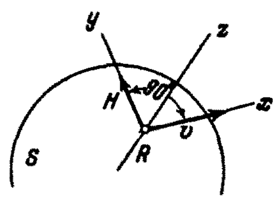

Fig. 2.

From the properties of a space gyrocompass it follows that the x-axis must be directed along the velocity vector of the suspension point with respect to the fixed sphere S, and the z-axis along the radius of this sphere.*

Let ω' be the angular velocity vector of the sensing element with respect to the fixed sphere S (or the angular velocity vector of the co-

* It should be mentioned that a local vertical coincides with earth's radius only on the equator and on the poles (at the latitude of 45° the discrepancy reaches 7′). Thus, in general, the xy-plane would not be horizontal in the ordinary sense, and its proper name should be "the plane of the geocentric horizon".

ordinate system xyz with respect to any inertial coordinate system, say $\xi^*\eta^*\zeta^*$

It is seen from Figure 2 that the x, y and z-components of the vector ω are

$$\omega_x = 0, \qquad \omega_y = \frac{v}{R}, \qquad \omega_z = \frac{v}{\rho_g} = \Omega(t) \tag{3}$$

Thus, the x-component of ω is known and equals zero. The y-component could also be regarded as known; it is determined through (1) when the information on the value of the angle 2σ between the axes of spin of the gyroscopes is continuously supplied.

The vertical component z of ω (strictly speaking, the radial component), also depends on the geodesic radius of curvature* of the trajectory traced by the suspension point on the fixed sphere S; hence $\Omega(t)$ is an unknown function of time.

4. Let ξ, η, ζ be an orthogonal coordinate system with the origin at the suspension point, the ζ-axis along the radius of earth, the η-axis along the tangent to the meridian directed north, and the ξ-axis along the tangent to the parallel directed East.

Let u be the angular velocity vector of the coordinate system ξ, η, ζ, with respect to the inertial system $\xi^*\eta^*\zeta^*$. The ξ, η, and ζ components of u are expressed [7] by the familiar formulas

$$u_\xi = -\frac{V_N}{R}, \; u_\eta = \frac{V_E}{R} + U\cos\varphi, \qquad u_\zeta = \frac{V_E}{R}\tan\varphi + U\sin\varphi \tag{4}$$

Here ϕ is the latitude**, and V_N and V_E are the northern and the eastern components of the velocity of the suspension point with respect to the earth's surface.

After substituting [7] the equations

$$V_N = \frac{d\varphi}{dt}, \qquad V_E = R\cos\varphi\,\frac{d\lambda}{dt} \tag{5}$$

* Geodesic curvature at a given point of a curve lying on a surface equals the curvature of the projection of the curve on a plane tangent to the surface at the point under consideration.

** The latitude ϕ is the geocentric latitude of a point. It differs from a geographic latitude by the magnitude of the angle between the local vertical and the earth's radius arising from the earth's rotation.

in (4) the components of u will have the following form

$$u_\xi = -\frac{d\varphi}{dt}, \qquad u_\eta = \left(U + \frac{d\lambda}{dt}\right)\cos\varphi, \qquad u_\zeta = \left(U + \frac{d\lambda}{dt}\right)\sin\varphi \tag{6}$$

where λ is the longitude.

5. Let θ be the angle between the x-axis and the ξ-axis, where x belongs to the xyz system moving with the sensing element and ξ belongs to the ξ, η, ζ system moving with the earth. It is easily seen that the angle θ is the so-called angle of deviation of gyroscope [7]. It is obvious from Figure 3 that

$$\operatorname{tg}\vartheta = \frac{V_N}{RU\cos\varphi + V_E} \tag{7}$$

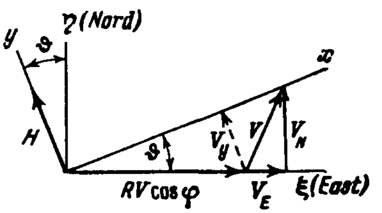

Fig. 3.

We repeat that the angle $\Omega(t)$ cannot be determined with the gyrocompass only. Formula (7) is not very convenient to determine the angle ϕ; we will therefore use an equivalent but more convenient formula

$$\sin\vartheta = \frac{V_y}{RU\cos\varphi} \tag{8}$$

Here V_y is a projection of velocity V on to the direction of the vector of kinetic moment H (y axis). In navigation, a log and a knowledge of the map ocean currents are essential for this purpose.

6. Moreover, the angular velocity vector u (of the geographic coordinate system ξ, η, ζ with respect to the fixed sphere S), and the angular velocity vector ω (of the coordinate system xyz moving with the sensing element) differ only in their ξ- and z-components (the axes ξ and z coincide). This difference (Fig. 3) is expressed by

$$\omega_z = u_z + \frac{d\vartheta}{dt} \tag{9}$$

From this statement it follows that

$$u_\xi = \omega_x\cos\vartheta - \omega_y\sin\vartheta, \qquad u_\eta = \omega_x\sin\vartheta + \omega_y\cos\vartheta, \qquad u_\zeta = \omega_z - \frac{d\vartheta}{dt} \tag{10}$$

Using formulas (3), (6), and (10), we obtain

$$\frac{d\varphi}{dt} = \frac{v}{R}\sin\vartheta, \quad \left(U + \frac{d\lambda}{dt}\right)\cos\varphi = \frac{v}{R}\cos\vartheta, \quad \left(U + \frac{d\lambda}{dt}\right)\sin\varphi = \Omega(t) - \frac{d\vartheta}{dt} \qquad (11)$$

If $\Omega(t)$ were a known function of time, then (11) would represent a system of nonlinear differential equations of the first order for the dependent variables ϕ, λ, and θ.

7. Determination of function $\Omega(t)$ requires in addition a directional gyroscope (Fig. 4), or some other gyroscopic device, like twin gyroscopes in a gyroazimuth (Fig. 5). In the case of a directional gyroscope we will use instantaneous values of the angle α between the x-axis (which moves with the sensing element), and the normal, ν, to the outer ring of the directional gyroscope (Fig. 6).

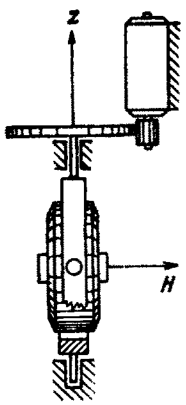

Fig. 4.

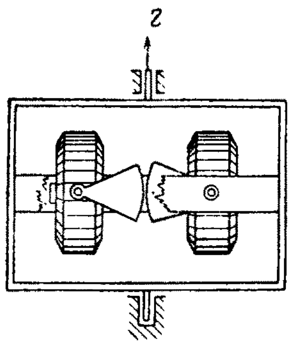

Fig. 5.

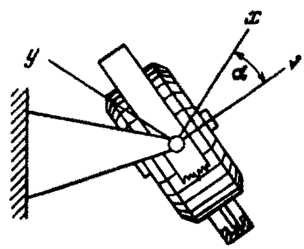

Fig. 6.

In the absence of friction in the casing axis of the directional gyroscope and the vanishing of the unbalancing moment of the system constituted by the casing plus spinning wheel about the axis of spin, the component of the angular velocity of the outer ring along the axis of the outer ring vanishes in any position of the directional gyroscope.

In a directional gyroscope the spin axis is always in a plane perpendicular to the plane of the outer ring, owing to a specially introduced moment about the axis of the inner ring. For our purposes the axis of the outer ring of the directional gyroscope should coincide with the radius of the "Earth" containing the suspension point of the sensing element of the space gyrocompass. This can be achieved through special recording devices which connect the sensing element of the space gyrocompass with the base of the directional gyroscope. Then the angular velocity of the sensing element and the angular velocity of the outer ring of the directional gyroscope will differ only in their z-components (that is in their components along the earth's radius).

Consequently we obtain

$$\omega_z = \Omega(t) = \frac{d\alpha}{dt} \tag{12}$$

and the function $\Omega(t) = \omega_z$ becomes known.

8. The system of differential equations (11) could be transformed into

$$\frac{d\varphi}{dt} = \frac{v}{R}\sin\vartheta, \qquad \frac{d\vartheta}{dt} = \frac{d\alpha}{dt} - \frac{v}{R}\cos\vartheta\,\mathrm{tg}\,\varphi, \qquad \frac{d\lambda}{dt} = \frac{v}{R}\frac{\cos\vartheta}{\cos\varphi} - U \tag{13}$$

where each equation is solved for the time derivative of the unknown dependent function ϕ, θ, and λ respectively.

The first two equations in the above system can be integrated independently of the third one. Hence, with initial conditions for ϕ and θ, the corresponding integrating device will continuously compute the latitude and the velocity correction for the gryocompass, and in this way the direction of the meridian is obtained without knowing the velocity of the suspension point with respect to the earth's surface.

When ϕ and θ have been computed, the last equation in (13) is easily integrated, yielding the latitude λ as the function of the initial latitude.

Thus the problem of the autonomous determination of position of a moving object is solved.

9. A space gyrocompass will obey the laws stated in [1], if the initial conditions are exactly those given there. Suppose that these conditions are not exactly satisfied. It was shown in [2] that in such a case the sensing element of a gyrocompass would perform small oscillations about the axes of the Darboux natural coordinate system $x^0y^0z^0$ in which the x^0-axis is along the velocity vector v of the suspension point with respect to the fixed sphere S, the z^0-axis along earth's radius, and the y^0 follows from the orthogonality.

Besides, the angle 2ϵ between the spin axes of the gyroscopes in the sensing element would also perform small oscillations about the value 2σ as given by formula (1). All these oscillations could be determined with great accuracy through superposition of two simple harmonic oscillations with frequencies $\sqrt{g/R} + U\sin\phi$. ($U\sin\phi$ is the vertical component of earth's angular velocity). Geckeler [8] mistakenly assumed that these frequencies equal so-called Schuller's frequency $\sqrt{g/R} = 0.00124065\ \mathrm{sec}^{-1}$.

It can be shown that while the oscillations of the z axis (moving with the sensing element of the space gyrocompass) have little influence on errors in angle α, the torsional vibrations of the sensing element about the z axis affect α considerably.

Thus, the integrating device, instead of receiving the true values of $\Omega(t)$ and v, receives data determined from the actual angle between the spin axes of the gyroscopes in the space gyrocompass according to formula (1), and from the instantaneous angle between the compass and the gyro-azimuth. An estimate of errors in coordinates of a moving object as obtained in this case from the integration of the equations (10) is an important practical problem deserving a special investigation.

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